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EFFECTS OF INTERNAL CONDUCTION ON THE DETERMINATION OF HEAT TRANSFER RATES USING THIN FILM MODELS

P. H. OOSTHUIZEN
Department of Mechanical Engineering, Queen's University, Kingston, Ontario, Canada

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NOMENCLATURE

D, diameter of cylinder;
G_D, Grashof number;
h, local heat transfer coefficient;
k_f, conductivity of fluid;
k_m, conductivity of core material;
N_D, actual local Nusselt number;
N_{D0}, local Nusselt number ignoring conduction;
q, rate of heat generation per unit area in surface film;
r, radial co-ordinate;
$\bar{r}$, r/R;
R, radius of cylinder;
T, temperature;
$\bar{T}$, dimensionless temperature, see equation (1);
T_s, local surface temperature;
$\bar{T}_s$, dimensionless local surface temperature;
T_f, fluid temperature;
ϕ, angular co-ordinate measured from stagnation point.

INTRODUCTION

LOCAL convective heat transfer rates have been determined in a number of studies using models in which the heat is generated electrically in a thin film, usually a thin metal sheet, on the surface of the model. In such models, the film is usually bonded to a core which is made from a material with a low thermal conductivity. A series of thermocouples are mounted in this core with their junctions on the surface of the core just below the film and with their leads brought out internally through the core. These thermocouples allow the surface temperature distribution to be measured and, since the electrical power dissipated in the surface film is also easily measured, it is possible to calculate the distribution of the local coefficient of heat transfer about the surface of the model. In this calculation, it is usual to ignore internal conduction of heat through the core from one part of the surface to another due to the low conductivity of the core material.

A model of the type described above was used, for example, in [1] in a study of the effects of turbulence on the heat transfer distribution about a circular cylinder in an air-stream, the Reynolds numbers used being relatively low. The heat transfer distribution obtained in this study was somewhat different from that obtained in previous studies using models of a different type and the authors' remark in [1] that "the variation in heat transfer is not as great as reported elsewhere".

A thin film type model was also used in [2] to study the effects of buoyancy forces on the distribution of the heat transfer rate about circular cylinders in low Reynolds number combined convective air flow. The distribution of heat transfer rate obtained in this study was in poor agreement with that obtained in [3] using a different type of model and with that predicted analytically in [4].

The purpose of the present note is to indicate that the disagreement between the results obtained in [1] and [2] using thin film models and those obtained in other studies is the result of internal conduction of heat through the core of the model from one part of the surface to another, this internal conduction being significant despite the low conductivity of the core material. It will be shown that when this internal conduction is accounted for using numerical methods, the results of [1] and [2] are brought into much closer agreement with those of earlier studies.

METHOD

Attention will be restricted in the present discussion to cylindrical bodies and it will be assumed that the temperature distribution is two-dimensional. The following dimensionless temperature and radius are defined.

$$\bar{T} = \left(\frac{k_m}{qR}\right)(T - T_f) \qquad \bar{r} = \frac{r}{R}. \tag{1}$$

In the present study, the thickness of the surface film and the temperature changes across it have been neglected. Due to its actual finite thickness, there is usually some circumferential conduction within the film. However, this effect is usually relatively small compared to the effects of internal conduction and it will, therefore, be neglected in the present work and q will be treated as a constant.

In terms of the variables defined in equation (1), the equation governing the temperature distribution within the core of the model becomes

$$\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{1}{\bar{r}^2}\frac{\partial^2 \bar{T}}{\partial \phi^2} = 0. \tag{2}$$

In [1] and [2], the variation of the local Nusselt number calculated by ignoring internal conduction i.e. of

$$N_{D0} = qD/(T_s - T_f)k_f \tag{3}$$

around the surface of the model are presented. The subscript 0 on N_{D0} indicates that it is the value of Nusselt number obtained by ignoring internal conduction within the model.

Now a comparison of equations (1) and (3) gives

$$\bar{T}_s = \left(\frac{2}{N_{D0}}\right)\left(\frac{k_m}{k_f}\right). \tag{4}$$

$\bar{T}_s$ being, of course, the dimensionless surface temperature. Therefore, the variation of $\bar{T}_s$ about the model surface can be deduced from the results given in [1] and [2] provided (k_m/k_f) is known. This variation provides one boundary condition subject to which equation (2) must be solved. Another set of boundary conditions follows from the fact that the temperature distribution is symmetrical so that $\partial T/\partial \phi$ is zero at $\phi = 0$ and at $\phi = \pi$. The last boundary condition subject to which equation (2) must be solved depends on whether the core of the model is completely solid or whether it has an axial hole down the centre. The model used in [1] was solid cored and for it the boundary condition is, of course, $\partial T/\partial r = 0$ at $r = 0$. The model used in [2] had a hole down the centre which was used to bring the thermocouples out of the model. If it is assumed that the heat transfer to the air in this hole is negligible, the boundary condition in this case is $\partial T/\partial r = 0$ at $r = r_i$, r_i being the radius of the core hole.

Equation (2) is easily solved subject to the above boundary conditions using standard numerical methods to give the variation of $\bar{T}$ with $\bar{r}$ and ϕ within the model.

Now a balance between the rate at which heat is generated locally at the surface and the rate at which it is transferred from the surface by conduction to the interior of the model and by convection to the fluid flowing over it gives

$$\left.\frac{\partial \bar{T}}{\partial \bar{r}}\right|_s + \left(\frac{hR}{k_m}\right)\bar{T}_s = 1 \tag{5}$$

where the subscript s again denotes conditions at the surface. Equation (5) can be used to determine the distribution of the local Biot number, i.e. (hR/k_m), about the model surface from the numerically calculated variation of $\bar{T}$ within the body. Using this variation, the variation of the local Nusselt number, N_D, can be found since, of course

$$N_D = 2\left(\frac{hR}{k_m}\right)\left(\frac{k_m}{k_f}\right). \tag{6}$$

RESULTS

The core of the model used in [1] was made from epoxy resin. This material can have a conductivity of from 10 to 50 times that of the air in which the tests were undertaken. Since the exact value for the resin from which the model was made was not known, calculations were carried out for (k_m/k_f) of 50 and 10. Typical of the results obtained are those shown in Fig. 1. Also shown in this figure are some of the results obtained in [5] for similar conditions using another type of model. It will be seen from these results that internal conduction effects are very significant and that when they are accounted for, the results of [1] are in fair agreement with those obtained in previous studies. The negative values of N_D obtained with (k_m/k_f) of 50 are, presumably, indicative of the fact that the material from which the model core was made did not have as high a value of k_m as indicated by this value.

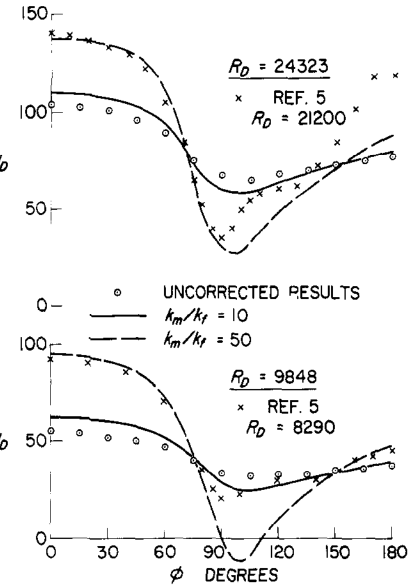

FIG. 1. Effect of internal conduction on some Nusselt number distributions obtained in [1].

The model used in [2] had a core made of plexiglas which has a conductivity that gives a (k_m/k_f) of about 7·5. Typical of the results obtained with this model when conduction is accounted for is that shown in Fig. 2. The variation obtained in [3] for similar conditions using a different type of model is also shown in Fig. 2. Again it will be seen that conduction effects are significant and that when they are accounted for, much closer agreement with previous results is obtained.

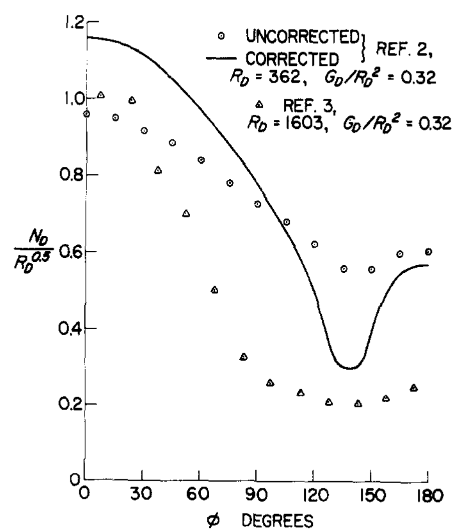

FIG. 2. Typical effect of internal conduction on results of [2].

ACKNOWLEDGEMENTS

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